Advanced Econometrics II
Large Programming Assignment Part 2

Deadline: 15.02.2015, 23:59

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Please submit your (typed) solution in a `pdf` file. **Please motivate all your answers.** The code has to be put, **together with** the main `pdf` solution file, in an archive file (e.g. `zip` or `rar`). Each code file shall contain your name.

Question 1

Consider the nonlinear regression model

$$y_t = \underbrace{\frac{\exp(\alpha t - \beta)}{1 + \exp(\alpha t - \beta)}}_{f_t(\alpha,\beta)} + \varepsilon_t,$$
$$\varepsilon_t \overset{iid}{\sim} \mathcal{N}(0, \sigma^2),$$

for $t = 1, \ldots, n$.

1° *(Nonlinear ML estimation)*
For the dataset `Data2.csv` obtain the ML estimate for the parameters vector $\theta = (\alpha, \beta)^T$, together with the corresponding covariance matrix estimate, using the **Newton-Raphson algorithm**, which in general is given by

1. Specify starting value θ_0 and set $k = 1$.
2. Repeat

$$\theta_k = \theta_{k-1} - \underbrace{\left(\frac{\partial^2 \ln g(y, \theta)}{\partial \theta} \bigg|_{\theta = \theta_{k-1}} \right)}_{H(\theta_{k-1})}^{-1} \frac{\partial \ln g(y, \theta)}{\partial \theta} \bigg|_{\theta = \theta_{k-1}},$$
$$k = k + 1,$$

until convergence (e.g. $||\theta_k - \theta_{k-1}|| < \epsilon$).

For convenience, you can apply the **Berndt-Hall-Hall-Hausman** (BHHH) method, where the Hessian matrix $H(\theta)$ is approximated with the outer product of gradients as follows

$$H(\theta) \approx \sum_{t=1}^{n} \left(\frac{\partial \ln g(y_t, \theta)}{\partial \theta} \right) \left(\frac{\partial \ln g(y_t, \theta)}{\partial \theta} \right)^T.$$

Hint: Here the whole procedure can be expressed for instance as follows:

1. Specify starting value θ_0 and set $k = 1$.
2. Repeat

$$\theta_k = \theta_{k-1} + \left(\dot{X}(\theta_{k-1})^T \dot{X}(\theta_{k-1})\right)^{-1} \dot{X}(\theta_{k-1})^T \left(y - x(\theta_{k-1})\right),$$
$$k = k + 1,$$

   until $\max |\theta_k - \theta_{k-1}| < 0.0001$,

where $y = (y_1, \ldots, y_n)^T$, $x(\theta) = (x_1(\theta), \ldots, x_n(\theta))^T$ with $x_t(\theta) = f_t(\alpha, \beta)$ and $\dot{X}(\theta) = \left(\dot{X}_1(\theta)^T, \ldots, \dot{X}_n(\theta)^T\right)^T$ with

$$\dot{X}_t(\theta) = \left(\frac{\partial f_t(\alpha,\beta)}{\partial \alpha}, \frac{\partial f_t(\alpha,\beta)}{\partial \beta}\right).$$

2° *(MLE finite sample properties)*
After you have estimated the model, compare the finite sample distribution of the MLE with its asymptotic distribution, where you assume $\varepsilon \overset{iid}{\sim} \mathcal{N}(0,1)$. For this purpose, perform Monte Carlo simulation to determine

i) the RMSE,

ii) the variance,

iii) the bias

of the estimator. In your simulations use the parameter values close to the values you have estimated in 1° for the provided dataset. Moreover, generate samples of length $n = 100$ (as in the dataset), $n = 1000$ and $n = 10000$. Compare and explain the results.