Advanced Econometrics II
Homework Assignment No. 6

Deadline: 16.02.2015, 23:59

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Please submit your (typed) solution in a `pdf` file. **Please motivate all your answers.** For programming exercises (if there are any), the code has to be put, together with the main `pdf` solution file, in an archive file (e.g. `zip` or `rar`). Each code file shall contain your name, either as a comment or in its name.

Question 1

Consider the loglikelihood function

$$l(y, \beta) = \sum_{t=1}^{n} (y_t \log F(X_t\beta) + (1 - y_t) \log(1 - F(X_t\beta))) \tag{11.09}$$

for a **binary response model**. Its maximization is easy when this function is globally concave, as then the FOCs, in general, uniquely define the MLE.

(a) *(Global concavity)*
Show that the contribution made by observation t to (11.09), and hence (11.09) itself, is **globally concave** with respect to β if the function F is such that $F(-x) = 1 - F(x)$, and if F, its derivative f, and its second derivative f' satisfy the condition

$$f'(x)F(x) - f^2(x) < 0, \qquad \forall x \in \mathbb{R}. \tag{11.88}$$

(b) Show that condition (11.88) is satisfied by the **logistic function** $\Lambda(\cdot)$, defined in

$$\Lambda(x) \equiv \frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x}, \tag{11.07}$$

(c) Show that condition (11.88) is satisfied by and the standard normal CDF $\Phi(\cdot)$.

Question 2

Consider two basic models for **more than two discrete responses**: the multinomial logit model and the ordered probit model.

(a) *(Multinomial logit model)*
Consider the special case when the explanatory variables are the same for each choice, i.e. where

$W_{tj} = X_t, \ \forall j$.
Construct the the loglikelihood function. Discuss the identification problem and how to proceed with estimation in this case.

(b) *(Ordered probit model)*
Construct the the loglikelihood function. Discuss the identification problem and how to proceed with estimation in this case.

Question 3

Consider a count variable y_t, $t = 1, \ldots, n$, with conditional mean

$$\mathbb{E}[y_t] = \exp(X_t\beta)$$

and conditional variance

$$\mathbb{E}[y_t - \exp(X_t\beta)] = \gamma 2 \exp X_t\beta.$$

We would like to apply the **Poisson regression model** for count data y, however we are rethinking this idea because of a possible **overdispersion** problem.

(a) *Sources of overdispersion*
Discuss what the overdispersion problem consists in and why it occurs in the Poisson model.

(b) *Testing for overdispersion*
Derive a possible test for overdispersion together with its asymptotic distribution.

(c) *(Asymptotic efficiency)*
Show that ML estimates of β under the incorrect assumption that y_t is generated by a Poisson regression model with mean $\exp(X_t\beta)$ are asymptotically efficient in this case.