

Advanced Econometrics II

Homework Assignment No. 5

Deadline: 09.02.2015, 23:59

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Please submit your (typed) solution in a pdf file. **Please motivate all your answers.** For programming exercises (if there are any), the code has to be put, together with the main pdf solution file, in an archive file (e.g. zip or rar). Each code file shall contain your name, either as a comment or in its name.

Question 1

Suppose that we are interested in testing some r restrictions $r(\theta) = 0$. For that purpose we consider **Wald test**, as we know that the principle for this test for models estimated by ML and for models estimated by other methods is the same.

(a) (*Asymptotic distribution*)

Explicitly derive the Wald test statistic for the null hypothesis and derive its asymptotic distribution under the null.

(b) (*Non-invariance*)

Provide an example to show that this test is not invariant to the way the null hypothesis is formulated.

Next, we want to apply the other two classical tests, typical for ML estimation: the **likelihood ratio** and **Lagrange multiplier** test. For simplicity, suppose we want to test the null $H_0 : \beta_2 = 0$. in the linear regression model

$$y = X_1\beta_1 + X_2\beta_2 + u, \quad u \sim N(0, \sigma^2 I),$$

where X_1 is $n \times k_1$, X_2 is $n \times k_2$, with $k_1 + k_2 = k$.

(c) (*LR test statistics*)

Explicitly derive the LR statistics for the null.

(d) (*LM test statistics*)

Explicitly derive the LM statistics for the null.

Question 2

Consider the following standard result for **transformations of variables**. Let X and Z be two random variables with cdfs F_X and F_Z , and pdfs f_X and f_Z , respectively. Suppose that they are related to each other by the equation $Z = h(X)$, where h is strictly increasing deterministic function. Then

$$f_X(x) = f_Z(h(x))h'(x).$$

Below you are asked to generalize and apply this result.

(a) (*Strictly monotonic functions*)

Let the two random variables X and Z be related by the deterministic equation $Z = h(X)$, where h is strictly decreasing. Show that the PDFs of the two variables satisfy the equation

$$f_X(x) = -f_Z(h(x))h'(x).$$

Then show that

$$f_X(x) = f_Z(h(x))|h'(x)|$$

holds whenever h is either strictly increasing or strictly decreasing.

(b) (*Different support*)

Let $X = Z^2$. Express the density of X in terms of that of Z , taking account of the possibility that the support of Z may include negative as well as positive numbers.

(c) (*Loglikelihood for a transformed model*)

Consider the model

$$y_t^{1/2} = X_t\beta + u_t, \quad u_t \sim NID(0, \sigma^2),$$

in which it is assumed that all observations y_t on the dependent variable are positive. Write down the loglikelihood function for this model.