Advanced Econometrics II
Homework Assignment No. 2

Deadline: 19.01.2015, 23:59

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Please submit your (typed) solution in a `pdf` file. Please motivate all your answers. For programming exercises (if there are any), the code (preferably a Matlab one) has to be put, together with the main solution file, in an archive file (e.g. `zip` or `rar`). Each code file shall contain your name, either as a comment or in its name.

Question 1

In this question we deal with the problem of testing the **overidentifying restrictions**[1]. For this purpose, consider the model

$$y = X\beta + u, \quad u \sim IID(0, \sigma^2 I),$$

where X is an $n \times k$ matrix of regressors, some of which are correlated with the error terms, and W is an $n \times l$, with $l > k$ matrix of instruments (overidentification). The IV estimation based on W can solve the problem of correlation between X and u if the instruments are valid, for which a sufficient condition is

$$\mathbb{E}(u_t | W_t) = 0.$$

Hence, we require all l instruments to be valid, but the 2SLS estimator effectively uses only k instruments $P_W X$.

(a) *(Extra instruments)*
How would you characterise the extra $l - k$ instruments? Are they uniquely determined? How would you use them to test whether $l - k$ overidentifying restrictions are valid?

(b) *(Test based on the IVGNR)*
Show how to use the extra instruments to construct an artificial IVGNR F test to test the overidentifying restrictions. That is, formulate the null and the alternative, give the formula for the test statistic and derive its asymptotic distribution under the null. Explain why you actually do not need to construct these extra instruments explicitly.

(c) *(Test based on the IV criterion function)*
Testing for the overidentifying restriction one can alternatively be based on the IV criterion function, which leads to the **Sargan test**. The test statistic for this test it the minimised IV criterion function for the model under the null divided by the IV estimate of the error variance for that model.

[1] Cf. Section 8.6 in DM.

Give the formula for its test statistics. Show that it is numerically identical to the F-test statistic based on the related IVGNRs – this will give you the asymptotic distribution of the Sargan test statistic under the null.

Hint: show that nR^2 from the modified IVGNR is actually equal to the Sargan test statistic.

(d) *(Interpretation of the test results)*
What is the interpretation of the Sargan test results? That is, what can you conclude when the Sargan test statistic is significant?

Question 2

This question concerns testing for **exogeneity** of the regressors[2], i.e. testing whether all the regressors are uncorrelated with the error terms. If this is the case, the OLS estimator is consistent and more efficient than the GIV estimator. Otherwise, only the GIV estimator is consistent.

(a) *(DWH test)*
Explain the **Durbin-Wu-Hausman** testing principle and derive the formula for the corresponding DWH test statistic.

(b) *(Equality of covariance matrices)*
Prove the Hausman result

$$\Big(\mathrm{Var}(\hat{\beta}_{IV} - \hat{\beta}_{OLS})\Big)^{-1} = \Big(\mathrm{Var}(\hat{\beta}_{IV}) - \mathrm{Var}(\hat{\beta}_{OLS})\Big)^{-1}.$$

Hint: Consider the OLS estimator and the **vector of contrasts**, i.e. $\hat{\beta}_{IV} - \hat{\beta}_{OLS}$. Derive the expected values of both vectors and consider their covariance matrix. Then, use the standard formula for the variance of a sum of arbitrary random variables. Derive the result under the null, so when the OSL estimator is unbiased and, when necessary, condition on X.

(c) *(Asymptotic distribution)*
Use the above result to derive the asymptotic distribution of the DWH test statistic under the null.

(d) *(Interpretation of the test results)*
What is the interpretation of the DWH test results? That is, what can you conclude when the DWH test statistic is significant?

Question 3

Finally, we want to put the two tests discussed above into practice. To this end, do Exercise 8.28 from DM, where you compare the OLS and 2SLS estimation results.

[2] Cf. Section 8.7 in DM.